Using the given matrix-chain <5, 10, 3, 12, 5, 50, 6>

A1 = 5, 10

A2 = 10, 3

A3 = 3, 12

A4 = 12, 5

A5 = 5, 50

A6 = 50, 6

p0=5, p1=10, p2=3, p3=12, p4=5, p5=50, p6=6

m[i, j] = 0, if i = j,

m[i,j]= {min {m[i,k] + m[k+1, j] + pi –1pkpj}}, if i < j

m[1,1] = m[2,2] = m[3,3] = m[4,4] = m[5,5] = m[6,6] = 0

m[1,2] = p0xp1xp2 = 5x10x3 = 150

m[2,3] = p1xp2xp3 = 10x3x12 = 360

m[3,4] = p2xp3xp4 = 3x12x5 = 180

m[4,5] = p3xp4xp5 = 12x5x50 = 3000

m[5,6] = p4xp5xp6 = 5x50x6 = 1500

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| m |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 |  | 0 |  | 150 |  | 330 |  | 405 |  | 1655 |  | 2010 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  | 0 |  | 360 |  | 330 |  | 2430 |  | 1950 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  | 0 |  | 180 |  | 930 |  | 1770 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  | 0 |  | 3000 |  | 1860 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  | 0 |  | 1500 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |  | 0 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

m[i,j]= {min {m[i,k] + m[k+1, j] + pi –1pkpj}}, if i < j

P0=5, p1=10, p2=3, p3=12, p4=5, p5=50, p6=6

Min of m[1,3] = 330

At k=1: m[1,3] = {min {m[1,1] + m[2,3] + p0p1p3}} = 0 + 10x3x12 + 5x10x12 = 0+360+600= 960

At k= 2: m[1,3] = {min{m[1,2] + m[3,3] + p0p2p3}} = 150 + 0 + 5x3x12 = 150+0+180= 330 (min)

Min of m[1,4] = 405; m[2,4] = 330

At k=1: m[1,4] = {min {m[1,1] + m[2,4] + p0p1p4}} = 0 + 330 + 5x10x5 = 330 + 250 = 580

m[2,4] = {min {m[2,2] + m[3,4] + p1p2p4}} = 0 + 180 + 10x3x5 = 180 + 150 = 330 (min)

m[2,4] = {min {m[2,3] + m[4,4] + p2p3p4}} = 360 + 3x12x5 = 360 + 180 = 540

At k=2: m[1,4] = {min {m[1,2] + m[3,4] + p0p2p4}} = 5x10x3 + 3x12x5 + 5x3x5 = 150+180+75 = 405 (min)

At k=3: m[1,4] = {min {m[1,3] + m[4,4] + p0p3p4}} = 330 + 0 + 5x12x5 = 330 + 300 = 630

Min of m[1,5] = 1655; m[3,5] = 930; m[2,5] = 2430

At k=1: m[1,5] = {min {m[1,1] + m[2,5] + p0p1p5}} = 0 + 2430 + 5x10x50 = 2430 + 2500 = 4930

For m[2,5] = {min {m[2,2] + m[3,5] + p1p2p5}} = 0 + 930 + 10x3x50 = 930 + 1500 = 2430 (min)

For m[3,5] = {min {m[3,3] + m[4,5] + p2p3p5}} = 0 + 3000 + 3x12x50 = 3000 + 1800 = 4800

For m[3,5] = {min {m[3,4] + m[5,5] + p2p4p5}} = 180 + 0 + 3x5x50 = 180 + 750 = 930 (min)

At k= 2: m[1,5] = {min{m[1,2] + m[3,5] + p0p2p5}} = 150 + 930 + 5x3x50 = 1080 + 750 = 1830

At k= 3: m[1,5] = {min{m[1,3] + m[4,5] + p0p3p5}} = 330 + 3000 + 5x12x50 = 3330 + 3000 = 6330

At k= 4: m[1,5] = {min{m[1,4] + m[5,5] + p0p4p5}} = 405 + 0 + 5x5x50 = 405 + 1250 = 1655 (min)

Min of m[1,6] = 2010; m[4,6] = 1860; m[3,6] = 1770; m[2,6] = 1950

At k=1: m[1,6] = {min {m[1,1] + m[2,6] + p0p1p6}} = 0 + 1950 + 5x10x6 = 1950 + 300 = 2250

For m[2,6] = {min {m[2,2] + m[3,6] + p1p2p6}} = 0 + 1770 + 10x3x6 = 1770 + 180 = 1950 (min)

For m[2,6] = {min {m[2,3] + m[4,6] + p1p3p6}} = 360 + 1860 + 10x12x6 = 2220 + 720 = 2940

For m[2,6] = {min {m[2,4] + m[5,6] + p1p4p6}} = 330 + 1500 + 10x5x6 = 1830 + 300 = 2130

For m[2,6] = {min {m[2,5] + m[6,6] + p1p5p6}} = 2430 + 0 + 10x50x6 = 2430 + 3000 = 5430

For m[3,6] = {min {m[3,3] + m[4,6] + p2p3p6}} = 0 + 1860 + 3x12x6 = 1860 + 216 = 2076

For m[3,6] = {min {m[3,4] + m[5,6] + p2p4p6}} = 180 + 1500 + 3x5x6 = 1680 + 90 = 1770 (min)

For m[3,6] = {min {m[3,5] + m[6,6] + p2p5p6}} = 930 + 0 + 3x50x6 = 930 + 900 = 1830

For m[4,6] = {min {m[4,4] + m[5,6] + p3p4p6}} = 0 + 1500 + 12x5x6 = 1500 + 360 = 1860 (min)

For m[4,6] = {min {m[4,5] + m[6,6] + p3p5p6}} = 3000 + 0 + 12x50x6 = 3000 + 3600 = 6600

At k= 2: m[1,6] = {min{m[1,2] + m[3,6] + p0p2p6}} = 150 + 1770 + 5x3x6 = 1920 + 90 = 2010 (min)

At k= 3: m[1,6] = {min{m[1,3] + m[4,6] + p0p3p6}} = 330 + 1860 + 5x12x6 = 2190 + 360 = 2550

At k= 4: m[1,6] = {min{m[1,4] + m[5,6] + p0p4p6}} = 405 + 1500 + 5x5x6 = 1905 + 150 = 2055

At k= 5: m[1,6] = {min{m[1,5] + m[6,6] + p0p5p6}} = 1655 + 0 + 5x50x6 = 1655 + 1500 = 3155

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| m |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 |  | 0 |  | 150 (1) |  | 330 (k=2) |  | 405 (k=2) |  | 1655 (k=4) |  | 2010 (k=2) |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  | 0 |  | 360 (2) |  | 330 (k=2) |  | 2430 (k=2) |  | 1950 (k=2) |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  | 0 |  | 180 (3) |  | 930 (k=4) |  | 1770 (k=4) |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  | 0 |  | 3000 (4) |  | 1860 (k=4) |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  | 0 |  | 1500 (5) |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |  | 0 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

“Each entry s[i, j] records a value of k such that an optimal parenthesization of AiAi+1…j splits the product between Ak and Ak+1.” Therefore, we get the following s-table:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| s |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 |  | 0 |  | 1 |  | 2 |  | 2 |  | 4 |  | 2 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  | 0 |  | 2 |  | 2 |  | 2 |  | 2 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  | 0 |  | 3 |  | 4 |  | 4 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  | 0 |  | 4 |  | 4 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  | 0 |  | 5 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |  | 0 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

In our problem, n=6. The array s[1…6, 1…6] has been computed above.

s[1,6] = 2 (A1A2)(A3A4)(A5A6)

s[1,2] = 1 (A1A2)

s[3,6] = 4 (A3A4)(A5A6)

So the final multiplication sequence is (A1A2)(A3A4)(A5A6)